

## Phenomenology of the electron structure function<sup>\*</sup>

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**Abstract.** The advantages of introducing the electron structure function (ESF) in electron induced processes are demonstrated. Contrary to the photon structure function it is directly measured in such processes. At present energies, a simultaneous analysis of both the electron and the photon structure functions gives an important test of the experimentally applied methods. Estimates of the ESF at LEP momenta are given. At very high momenta contributions from  $W$  and  $Z$  bosons together with  $\gamma$ - $Z$  interference can be observed. Predictions for the next generation of experiments are given.

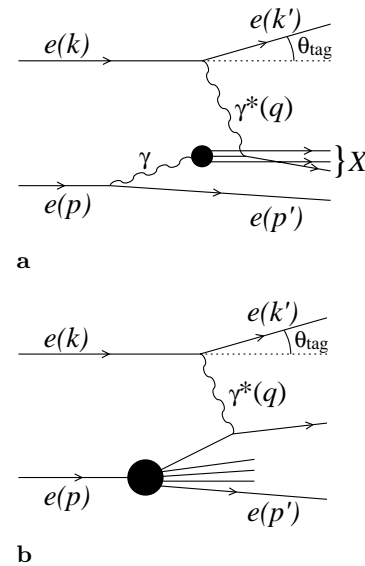
In a series of papers [1, 2] we have presented the construction of the electron structure function – a useful notion in the QCD analysis of electron induced hadron production. The  $Q^2$  evolution equations have been constructed and asymptotic solutions found for the quark and gluon content of the electron in the leading logarithmic approximation. We included contributions from all intermediate bosons; in particular, we found that the  $\gamma$ - $Z$  interference is important at very high energies. This spoils the usual probabilistic interpretation of separate  $\gamma/Z/W$  structure functions. We also found that in certain experimental situations the commonly used convolution of the photon structure function and the photon flux, used to describe the electron induced processes, is incorrect.

In this article we study the phenomenology of the electron structure function comparing it with the well-known approach which makes use of the photon structure function. The theoretical framework which allows one to calculate the photon structure is known since long [3]. It appears as a perturbative QCD contribution, in addition to the modeled vector meson dominance term. To measure this photonic structure, experiments [4] use the electron (or positron) beam as a source of photons. Despite precise measurements the photon structure function is not easy to extract. The problem is displayed in Fig. 1a. The tagged (upper) electron emits a probing photon, whereas the untagged (lower) one goes nearly along the beam, emitting the target photon. (The situation where at very high energies the probing boson can also be a  $Z$  boson is considered below.) The large scale  $Q^2$  is determined by the tagged electron:

$$Q^2 = -(k - k')^2 = 2EE_{\text{tag}}(1 - \cos \theta_{\text{tag}}), \quad (1)$$

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**Fig. 1a,b.** Deep inelastic scattering on a photon **a** and electron **b** target

where  $E$  is the initial electron energy and  $E_{\text{tag}}$  and  $\theta_{\text{tag}}$  are the energy and polar angle of the measured electron. The antitag condition (if present) requires the virtuality of the target photon  $P^2$  to be less than a certain  $P_{\text{max}}^2$ :

$$-(p - p')^2 \equiv P^2 \leq P_{\text{max}}^2. \quad (2)$$

This photon is clearly not a beam particle and has the energy diffused according to the equivalent photon spectrum:

$$f_{\gamma}^e(y_{\gamma}, P^2) = \frac{\alpha}{2\pi P^2} \left[ \frac{1 + (1 - y_{\gamma})^2}{y_{\gamma}} - \frac{2y_{\gamma}m_e^2}{P^2} \right], \quad (3)$$

where  $y_{\gamma}$  is the photon momentum fraction,  $\alpha$  is the QED structure constant, and  $m_e$  is the electron mass. The mea-

sured cross-section for the production of a hadronic system  $X$  (at  $Q^2 \gg P_{\max}^2$ ) expressed in terms of the photon structure functions  $F_2^\gamma$  and  $F_L^\gamma$  reads

$$\frac{d^3\sigma_{ee \rightarrow eX}}{dzdQ^2dx} = \frac{2\pi\alpha^2}{x^2Q^4} \int_{P_{\min}^2}^{P_{\max}^2} \left[ (1 + (1-y)^2)F_2^\gamma(x, Q^2, P^2) - y^2F_L^\gamma(x, Q^2, P^2) \right] f_\gamma^e(y_\gamma, P^2) dP^2, \quad (4)$$

where

$$y = 1 - (E_{\text{tag}}/E) \cos^2(\theta_{\text{tag}}/2), \quad (5)$$

$$P_{\min}^2 = m_e^2 y_\gamma^2 / (1 - y_\gamma)$$

and  $x$  ( $z$ ) are fractions of the parton momentum with respect to the photon (electron). They are related to the photon momentum fraction with respect to the electron by

$$z = xy_\gamma. \quad (6)$$

The integral over the photon virtuality is usually performed by assuming  $P^2 = 0$  in the photon structure functions (Weizsäcker–Williams approximation [7]) which leads to

$$\begin{aligned} \frac{d^3\sigma_{ee \rightarrow eX}}{dzdQ^2dx} &= \frac{2\pi\alpha^2}{x^2Q^4} \\ &\times [(1 + (1-y)^2)F_2^\gamma(x, Q^2, 0) - y^2F_L^\gamma(x, Q^2, 0)] \\ &\times f_\gamma^{\text{WW}}(z/x, P_{\max}^2), \end{aligned} \quad (7)$$

where

$$\begin{aligned} f_\gamma^{\text{WW}}(y_\gamma, P_{\max}^2) & \quad (8) \\ &= \frac{\alpha}{2\pi} \left[ \frac{1 + (1 - y_\gamma)^2}{y_\gamma} \ln \frac{P_{\max}^2(1 - y_\gamma)}{m_e^2 y_\gamma^2} - 2 \frac{1 - y_\gamma}{y_\gamma} \right]. \end{aligned}$$

This is how the real photon structure functions  $F_2$ ,  $F_L$  appear in lepton–lepton scattering.

Three remarks are important for further considerations. First, the splitting of the process into a distribution of photons inside the electron  $f_\gamma^{\text{WW}}$  and that of partons inside the photon  $F_2^\gamma$  is an approximation. The optimal form of the equivalent photon formula is still being discussed [8]. Even if most of experimental groups choose the same formula, one should keep in mind that the photon structure function depends on this convention.

Second, the target photon is always off-shell and its virtuality is experimentally not measured. Although the equivalent photon distribution is peaked at minimum (nearly zero) virtuality, treating the photon as real is another approximation. One should keep in mind that the measured photon structure function depends on  $x$ ,  $Q^2$  and  $P^2$ , and the, usually neglected,  $P^2$  dependence can be quite strong [12]. A hint that we are not measuring the real photon structure function comes also from the analysis of the QED structure function of the photon, where analytical solutions (and thus the  $P^2$  dependence) are known [5]. The data (coming from  $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$  scattering

[6]) agree with theory only if non-zero virtuality is taken into account. In the case of QCD the situation is more difficult – theory predicts a  $P^2$  dependence only in the perturbative region and the non-perturbative part has to be modeled phenomenologically.

Third, in order to fix  $x$ , one is forced to measure – in addition to the tagged electron – the hadronic momenta. In fact,

$$x = \frac{Q^2}{Q^2 + P^2 + W^2} \approx \frac{Q^2}{Q^2 + W^2}, \quad (9)$$

where  $W$  is the invariant mass of the produced hadronic system  $X$ . Its determination is more difficult than of other (tagged electron) variables, since a substantial part of the hadrons is lost in the beam pipe. The uncertainty in the determination of the  $x$  variable is the source of large uncertainties in the analysis (unfolding procedure). The data are indirectly biased by theoretical assumptions.

Many of the above problems can be avoided when we introduce the structure function of the electron (Fig. 1b). To see how it works let us first write the cross-section (at  $Q^2 \gg P_{\max}^2$ ) in terms of the electron structure functions  $F_2^e$  and  $F_L^e$ :

$$\begin{aligned} \frac{d^2\sigma_{ee \rightarrow eX}}{dzdQ^2} &= \frac{2\pi\alpha^2}{zQ^4} \left[ (1 + (1-y)^2)F_2^e(z, Q^2, P_{\max}^2) \right. \\ &\quad \left. - y^2F_L^e(z, Q^2, P_{\max}^2) \right]. \end{aligned} \quad (10)$$

The structure function  $F_2^e(z, Q^2, P_{\max}^2)$ , which dominates the cross-section at small  $y$ , has a simple partonic interpretation:

$$F_2^e(z, Q^2, P_{\max}^2) = z \sum_i e_{q_i}^2 q_i(z, Q^2, P_{\max}^2), \quad (11)$$

where  $e_{q_i}$  and  $q_i$  are the  $i$ th quark fractional charge and density. This is a standard deep inelastic scattering process where the cross-section is related to the structure function via simple kinematical factors. More precisely, the above defined electron structure function corresponds exactly to the proton structure function if no antitag condition is imposed and  $P_{\max}^2$  goes up to its kinematical limit of the order of  $Q^2$ . (This is the “inclusive case” in our terminology [1]; the electron structure function depends then on  $z$  and  $Q^2$  only.)

The argument  $z$  – the parton momentum fraction with respect to the electron – is measured, as in the standard deep inelastic scattering, by means of the tagged electron variables only:

$$z = \frac{Q^2}{2pq} = \frac{\sin^2(\theta_{\text{tag}}/2)}{E/E_{\text{tag}} - \cos^2(\theta_{\text{tag}}/2)}. \quad (12)$$

There is no need a priori to reconstruct the hadronic mass  $W$ . In present experimental analyses one introduces in both the photon and electron structure analyses a lower limit on  $W$  because the reconstruction of the hadronic mass is unreliable below this limit. But even so, there is an important difference between the case when  $W$  (in

the small  $W$  region) is used only as a selection cut to reduce background (electron structure) and the case when in addition it is used to determine the  $x$  variable (photon structure). In particular, a lower cut on  $W$  is much less sensitive to the unfolding procedure than the value of  $W$  itself. The quantitative analysis of the influence of  $W$  is discussed below.

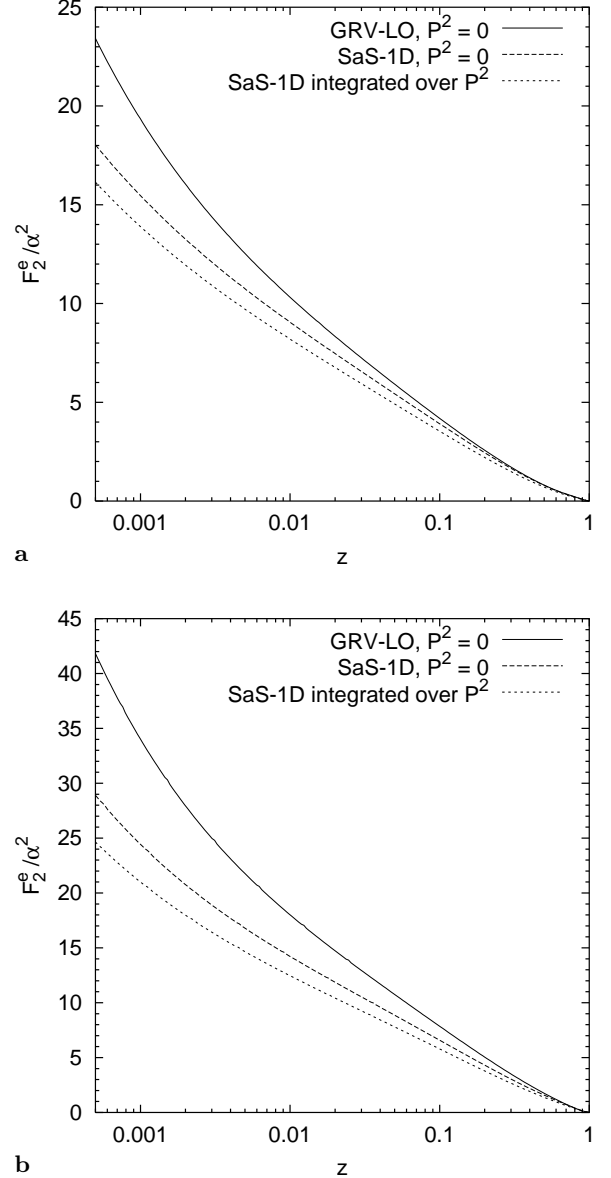
All these features are the cause that the same experiment can produce more precise and analysis independent data when looking at the electron structure. What is most important is that the electron structure function contains the same information about QCD as the photon one, and it is known theoretically with at least the same accuracy. Moreover, it allows one to avoid problems which arise in the photon structure function at very high energies.

At present energies, where the  $W$  and  $Z$  bosons contributions are negligible, one can reanalyse the existing data in terms of the electron structure function. This can be treated also as a consistency check of both photon and electron structure. Phenomenologically, having a parameterization of the photon structure function which describes well the existing data, we can predict the electron structure function for  $Q^2 \gg P_{\max}^2$  by taking the convolution of this parameterization with the equivalent photon spectrum:

$$F_2^e(z, Q^2, P_{\max}^2) = \int_z^1 dy_\gamma \int_{P_{\min}^2}^{P_{\max}^2} dP^2 f_\gamma^e(y_\gamma, P^2) \times F_2^\gamma\left(\frac{z}{y_\gamma}, Q^2, P^2\right). \quad (13)$$

Such a convolution is correct when the experiments use the antitag condition (“exclusive case” in our terminology [1]). The curves with momenta corresponding to LEP2 and TESLA/NLC/JLC experiments, resulting from some popular parameterizations [10,11] of the real photon structure, i.e.  $P^2 = 0$  in  $F_2^\gamma$  of (13), are shown in Fig. 2. To test how significant the (neglected above)  $P^2$  dependence is, we also plot the resulting curve of the SaS-1D parameterization of the virtual photon structure with the  $P^2$  dependence built in [11]. In the latter case the integration over  $P^2$  in (13) is performed numerically. Our previous arguments are confirmed: already at LEP2 the photon virtuality is non-negligible; it can produce effects of the same order as differences between various parameterizations. We stress once again that the electron variables  $z, Q^2$  and  $P_{\max}^2$  are well defined experimentally, whereas the photon virtuality is not measured.

As noticed in [13], the shape of the electron structure function is strongly influenced by the QED part (“photon flux”). All  $F_2^e$  functions resulting from various photon structure parameterizations are decreasing functions of  $z$  and look “similar”. In addition the contributions at a given electron momentum fraction  $z = xy_\gamma$  come both from large  $x$  and small  $y_\gamma$  as well as large  $y_\gamma$  and small  $x$ . Both these features can be regarded as drawbacks of the electron structure function. The above arguments are only partly true. First, one should keep in mind that the



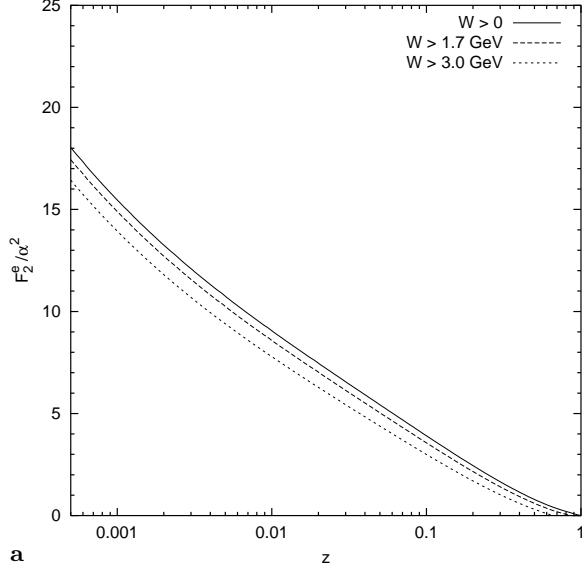
**Fig. 2a,b.** Predicted value of the electron structure function  $F_2^e(z)/\alpha^2$  at **a**  $Q^2 = 17.8 \text{ GeV}^2$ ,  $P_{\max}^2 = 4.5 \text{ GeV}^2$  and **b**  $Q^2 = 120 \text{ GeV}^2$ ,  $P_{\max}^2 = 30 \text{ GeV}^2$  from different parameterizations [10,11] of the photon structure function: SaS-1D (broken line), GRV-LO (solid line)

variables  $x, y_\gamma$  and  $z$  are not independent and their interplay under the integral (13) is specific. To see the problem in more detail let us take (13) within the Weizsäcker–Williams approximation ( $P_{\max}^2, P^2, Q^2$  suppressed):

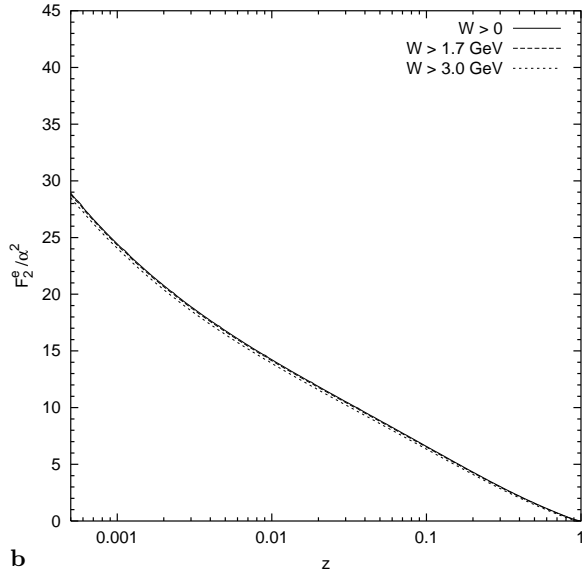
$$F_2^e(z) = \int_z^1 dy_\gamma f_\gamma^{\text{WW}}(y_\gamma) F_2^\gamma\left(\frac{z}{y_\gamma}\right). \quad (14)$$

Noting that essentially  $f_\gamma^{\text{WW}}(y_\gamma) \propto 1/y_\gamma$ , we get from (14)

$$F_2^e(z) \propto \int_z^1 \frac{dy_\gamma}{y_\gamma} F_2^\gamma\left(\frac{z}{y_\gamma}\right) = \int_z^1 \frac{dx}{x} F_2^\gamma(x). \quad (15)$$



a

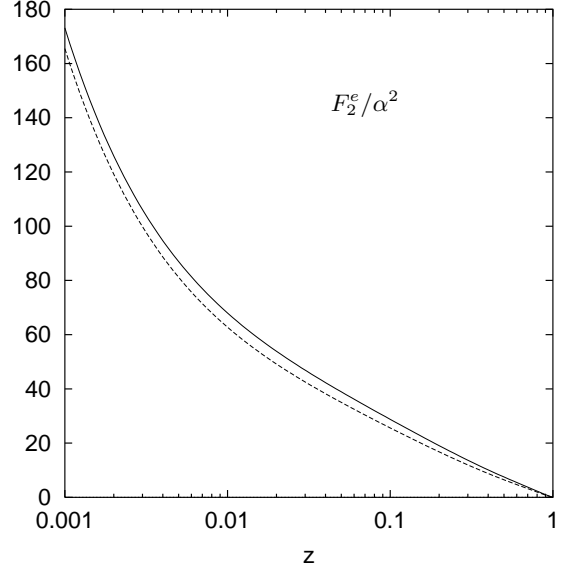


b

**Fig. 3a,b.** Predicted value of the electron structure function  $F_2^e(z)/\alpha^2$  from SaS-1D parameterization (solid line). The effect on measured values when a cut on  $W$  is imposed:  $W \geq 1.7$  GeV (dashed line),  $W \geq 3.0$  GeV (dotted line). **a**  $Q^2 = 17.8$  GeV<sup>2</sup>,  $P_{\max}^2 = 4.5$  GeV<sup>2</sup>; **b**  $Q^2 = 120$  GeV<sup>2</sup>,  $P_{\max}^2 = 30$  GeV<sup>2</sup>

One sees that in this approximation the  $z$  dependence comes via the lower limit of integration. Moreover, the importance of the small  $x$  region under the integral (14) is enhanced by the  $1/x$  factor. Second, due to the same kinematics the data points are generally shifted towards lower  $z$  (as compared to  $x$ ). Therefore the experimental results are more accurate at small  $z$ , a feature mostly welcome in the region where new effects are to be expected. In addition, we recall that the photon data points at low  $x$  are strongest influenced by the unfolding procedure.

As already mentioned, the present way of data analysis introduces a lower cut on the hadronic mass  $W$ . In the photon case it causes us not to be able to measure the pho-



**Fig. 4.** The structure function  $F_2^e(z, Q^2, P_{\max}^2)/\alpha^2$  at  $Q^2 = 50000$  GeV<sup>2</sup> and  $P_{\max}^2 = 1000$  GeV<sup>2</sup> (solid line). The contribution from the photon only is also shown (broken line)

ton structure function above a certain  $x$  (see (9)). In the measurement of the electron structure function  $W$  is not needed at all (as it is not used in the analysis of the proton structure function). A cut on  $W$  imposed in the present experiments lowers the cross-section in the whole  $z$  range, see Fig. 3. We checked<sup>1</sup> that e.g. for  $Q^2 = 17.8$  GeV<sup>2</sup> the effect of the condition  $W \geq 1.7$  GeV (with the SaS-1D parameterization in (13)) is less than 5% for  $z \leq 0.01$ , and less than 9% for  $z \leq 0.1$ . This suppression gets smaller with growing  $Q^2$  (e.g. at  $Q^2 = 120$  GeV<sup>2</sup> it is below 1% for  $z \leq 0.1$ ).

The concept of the electron structure function introduces new interesting effects at momenta much higher than presently available. One has then to take into account not only the photon flux contributing to the electron structure but also those resulting from  $Z$  and  $W$  bosons. As shown in [1]  $\gamma$ - $Z$  interference comes into play and is comparable to the  $Z$  contribution itself. This means that the notion of separate gauge boson structure functions ( $\gamma$ ,  $Z$  or  $W$ ) loses sense and only the electron structure function preserves a probabilistic interpretation. The question is: can we observe these effects in the next generation of experiments? In Fig. 4 we give a quantitative estimate in the case of single tag  $e^+e^-$  scattering at CLIC [9] momenta, choosing  $Q^2 = 10000$  GeV<sup>2</sup> and  $P_{\max}^2 = 1000$  GeV<sup>2</sup>. In this case the picture of Fig. 1 gets modified. The upper (tagged) electron emits now both the photon and the  $Z$  boson. In the calculation of the electron structure one has to take into account contributions from the photon, the  $Z$  boson, their interference (with the antitag condition fixed by  $P_{\max}^2$ ) and the  $W$  boson (no antitag condition). The presented curves are asymptotic solutions of our electron evolution equations [1]. One sees that the effect of  $Z$  and

<sup>1</sup> The selected  $Q^2$  values correspond to the latest measurements of the photon structure function at LEP

$\gamma$ - $Z$  terms is of the order of 5–15%. We checked that it can be enhanced to 20–25% in a double tag experiment.

Let us add a few final remarks. The first one concerns the study of the virtual photon structure [12] (double tag experiment). The analysis can be reformulated in terms of the  $P_{\max}^2$  dependence of the electron structure function. Studying a real, convention independent object is a first advantage. Another one is the fact that at very high virtualities the  $Z$  admixture and the  $\gamma$ - $Z$  interference are properly taken into account.

Second is a comment on the QED structure function of the photon. It is obtained from the process  $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$  by dividing out the (approximate) equivalent photon distribution and assuming some effective photon virtuality. The use of the QED electron structure function avoids the above approximations. The exactly known (in given order of  $\alpha$ ) electron structure function can be compared directly with the electron data.

Finally, the photon structure has been also measured [14] in dijet production at HERA. Again the extraction of the  $x$  variable is difficult. In addition to jets, one has to measure essentially the whole hadronic system in order to obtain the photon energy. The data, when presented in terms of the electron structure, require only measurement of the two jets. Practically the new approach means plotting the dijet cross-section as a function of  $z$ . A more ambitious program would be to extract the parton densities in the electron, as has been done in the case of the photon [15] or to construct a parameterization of parton densities inside the electron by a direct fit to the HERA data.

To summarise, we propose to look at the electron as surrounded by a QCD cloud of quarks and gluons (in order  $\alpha^2$ ), very much like it is surrounded by a QED cloud of equivalent photons (in order  $\alpha$ ). We argue that the use of the electron structure function in electron induced processes has some advantages over the photon one. Experimentally it leads to more precise, convention independent data. Theoretically it allows for a more careful treatment of all variables. It also takes into account all electroweak gauge boson contributions, including their interference, which will be important in the next generation of  $e^+e^-$  colliders. At present energies it should certainly be used as a cross-check of the photon structure analysis.

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